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Non-BPS solutions of the noncommutative CP^1 model in $2+1$ dimensions

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Abstract: We find non-BPS solutions of the noncommutative CP^1 model in $2+1$ dimensions. These solutions correspond to soliton anti-soliton configurations. We show that the one-soliton one-anti-soliton solution is unstable when the distance between the soliton and the anti-soliton is small. We also construct time-dependent solutions and other types of solutions.

Keywords: Sigma Models, Solitons Monopoles and Instantons, Non-Commutative Geometry.

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Contents

1. Introduction

Noncommutative field theories naturaly arise as low-energy descriptions of string theory (for a review see [1]). Non-perturbative dynamics of string theory was investigated through the study of solitons and instantons in noncommutative gauge theories (see e.g. [2]). In four-dimensional Yang-Mills theory, there exist instantons even in the U(1) case [3]. Non-trivial solutions are also known in noncommutative scalar theories [4]. The scattering of noncommutative solitons was studied in Yang-Mills theory [5] and in scalar theories [6, 7].

In lower dimensions, nonlinear sigma models exhibit many similarities with four-dimensional Yang-Mills theory. In $2+1$ dimensions, nonlinear sigma models possess soliton solutions. The BPS solitons of the CP^N model were extended to a noncommutative space [8]. The low-energy dynamics of the BPS solitons in the noncommutative CP^1 model was investigated [9]. In noncommutative integrable sigma models, time-dependent solutions were written down explicitly [10] and the scattering of solitons and anti-solitons was studied [11].

In this paper, we consider the noncommutative CP^1 model in $2+1$ dimensions. In the commutative CP^N model, general static solutions are known [12]. For $N \geq 2$, there exist non-BPS solutions in addition to the BPS solutions. On the other hand, in the commutative CP^1 model no non-BPS solutions exist. We construct non-BPS solutions of the noncommutative CP^1 model and study their dynamics. These solutions represent the co-existence of solitons and anti-solitons.

This paper is organized as follows. In the following section, we summarize the properties of the noncommutative CP^N model. In section 3, we construct non-BPS solutions of the noncommutative CP^1 model and investigate the stability of the solutions. We further construct time-dependent solutions by boosting static solutions. In section 4, other types of solutions are presented. Finally, in section 5 we discuss future problems.

2. The noncommutative CP^N model in $2+1$ dimensions

We consider the CP^N model on a (2+1)-dimensional noncommutative spacetime [8] whose spatial coordinates obey the commutation relation

$$[\hat{z}, \hat{\bar{z}}] = \theta > 0\,, \tag{2.1}$$

where $\hat{z} = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y})$, $\hat{\bar{z}} = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y})$. Since (2.1) is the commutation relation of creation and annihilation operators, we can identify the field with an operator in the Fock space of (2.1). The lagrangian of the model is given by

$$L = 2\pi\theta\,\mathrm{Tr}\left(|D_t\Phi|^2 - |D_z\Phi|^2 - |D_{\bar{z}}\Phi|^2\right), \tag{2.2}$$

where Φ is a $(N+1)$-component complex vector with the constraint $\Phi^\dagger\Phi = 1$, and Tr denotes the trace over the Fock space. The covariant derivative is defined by

$$D_a\Phi = \partial_a\Phi - i\Phi A_a\,, \qquad A_a = -i\Phi^\dagger\partial_a\Phi\,, \qquad a = t, z, \bar{z}\,, \tag{2.3}$$

where

$$\partial_z = -\theta^{-1}[\hat{\bar{z}},\ ]\,, \qquad \partial_{\bar{z}} = \theta^{-1}[\hat{z},\ ]\,. \tag{2.4}$$

The model has a global SU(N) symmetry and a local U(1) symmetry, $\Phi \to \Phi g$, $g \in \mathrm{U}(1)$.

The energy of a static configuration is given by

$$\begin{aligned} E &= 2\pi\theta\,\mathrm{Tr}(|D_z\Phi|^2 + |D_{\bar{z}}\Phi|^2) \\ &\geq 2\pi|Q|\,, \end{aligned} \tag{2.5}$$

where

$$Q = \theta\,\mathrm{Tr}(|D_z\Phi|^2 - |D_{\bar{z}}\Phi|^2) \tag{2.6}$$

is the topological charge. The configuration which saturates the energy bound satisfies the BPS soliton equation

$$D_{\bar{z}}\Phi = 0\,, \tag{2.7}$$

or the BPS anti-soliton equation

$$D_z\Phi = 0\,. \tag{2.8}$$

The BPS (anti-)soliton solution has the positive (negative) topological charge.

In order to find solutions of (2.7) and (2.8), it is convenient to introduce a (N+1)-component complex vector W and the projection operator P as

$$\Phi = W\frac{1}{\sqrt{W^\dagger W}}\,, \tag{2.9}$$

$$P = W\frac{1}{W^\dagger W}W^\dagger\,. \tag{2.10}$$

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P satisfies the relations

$$PW = W\,, \qquad P^\dagger = P\,, \qquad P^2 = P\,. \tag{2.11}$$

In terms of W and P (2.7) can be written as

$$(1-P)\partial_{\bar z} W = 0\,. \tag{2.12}$$

The general solution of the BPS soliton equation (2.12) is given by

$$W = W_0(\hat z)\Delta(\hat z, \hat{\bar z})\,, \tag{2.13}$$

where the components of $W_0(\hat z)$ is polynomials of $\hat z$ and $\Delta(\hat z, \hat{\bar z})$ is an arbitrary invertible scalar function. The highest degree of the components of $W_0(\hat z)$ is equal to the topological charge. The anti-soliton solution has the same form as (2.13) with the components of W_0 being polynomials of $\hat{\bar z}$.

In the CP^N model some gauge transformations become singular after the noncommutative extension. In the commutative CP^1 model $W_1 = \begin{pmatrix} \mu z^{-1} \\ 1 \end{pmatrix}$, where μ is a parameter, is gauge-equivalent to the BPS soliton solution $W_2 = \begin{pmatrix} \mu \\ z \end{pmatrix}$, but this is not true in the noncommutative case. In the noncommutative case $\hat z$ is not invertible because $\hat z$ has a zero eigen value. However we can define the right inverse element $\hat z^{-1} = \hat{\bar z}\frac{1}{\hat z \hat{\bar z}+\theta}$ satisfying $\hat z \hat z^{-1} = 1$ and $\hat z^{-1}\hat z = 1 - |0\rangle\langle 0|$. Since $\tilde W = \begin{pmatrix} \mu \hat z^{-1} \\ 1 \end{pmatrix}$ is not of the form (2.13), $\tilde W$ is not a BPS solution. Moreover, $\tilde W$ is not a solution of the equation of motion. In section 4 we use $\tilde W$ to construct a solution.

In terms of P the lagrangian is written as

$$L = \pi\theta\,\mathbf{Tr}(\partial_t P \partial_t P - 2\partial_{\bar z} P \partial_z P)\,, \tag{2.14}$$

where $\mathbf{Tr}$ consists of the trace over the Fock space and that over the $(N+1)\times(N+1)$ matrix indices. The equation of motion is

$$[\partial_t^2 P - 2\partial_{\bar z}\partial_z P, P] = 0\,. \tag{2.15}$$

For a static configuration this equation is written as

$$[[\hat z, [\hat{\bar z}, P]], P] = 0\,, \tag{2.16}$$

where we have used (2.4). The BPS equations (2.7) and (2.8) are written as

$$(1-P)\hat z P = 0\,, \tag{2.17}$$
$$(1-P)\hat{\bar z} P = 0\,. \tag{2.18}$$

Rewriting (2.16) as

$$[\hat{\bar z}, (1-P)\hat z P] + [\hat z, P\hat{\bar z}(1-P)] = 0\,, \tag{2.19}$$

we see that solutions of (2.17) satisfy (2.16). Similarly solutions of (2.18) satisfy (2.16).

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3. Non-BPS solutions of the noncommutative CP^1 model

In this section we consider non-BPS solutions of the noncommutative CP^1 model, i.e. solutions of (2.16) which satisfy neither (2.17) nor (2.18). These solutions correspond to soliton anti-soliton configurations. We investigate the stability of the one-soliton one-anti-soliton solution. We also construct the boosted solution.

3.1 Soliton anti-soliton solution

We consider the 2×2 projection operator of the diagonal form

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} . \tag{3.1}$$

If P satisfies the equation of motion (2.16), P_1 and P_2 also do. We take P_1 and P_2 to be solutions of (2.17) and (2.18) respectively and hence we have

$$(1 - P_1)\hat{z}P_1 = 0 \,, \tag{3.2}$$

$$(1 - P_2)\hat{\bar{z}}P_2 = 0 \,. \tag{3.3}$$

In this case P satisfies the equation of motion but not the BPS equations. In the commutative case there exist only the trivial solutions of (3.2) and (3.3), $P_{1,2} = 0, 1$, but in the noncommutative case non-trivial solutions are known. The solution of (3.2) is given by [6, 10, 13]

$$P_1 = \sum_{i,j=1}^{r} |z^i\rangle h^{-1}_{ij} \langle z^j| \,, \tag{3.4}$$

where

$$|z^i\rangle = e^{\theta^{-1}(z^i\hat{\bar{z}} - \bar{z}^i\hat{z})}|0\rangle \,, \tag{3.5}$$

$$h^{ij} = \langle z^i|z^j\rangle \,, \qquad h^{-1}_{ij} h^{jk} = \delta_i{}^k \,. \tag{3.6}$$

Since $1 - P_2$ satisfies the BPS soliton equation (2.17), P_2 can be written as

$$P_2 = 1 - \sum_{k,l=1}^{s} |\tilde{z}^k\rangle \tilde{h}^{-1}_{kl} \langle \tilde{z}^l| \,. \tag{3.7}$$

Therefore P takes the following form

$$P = \begin{pmatrix} \sum_{i,j=1}^{r} |z^i\rangle h^{-1}_{ij} \langle z^j| & 0 \\ 0 & 1 - \sum_{k,l=1}^{s} |\tilde{z}^k\rangle \tilde{h}^{-1}_{kl} \langle \tilde{z}^l| \end{pmatrix} . \tag{3.8}$$

To see that z^i $(i = 1, \ldots, r)$ and $\tilde{z}^k$ $(k = 1, \ldots, s)$ are interpreted as positions of solitons and anti-solitons respectively, we consider the large $|z^i|$ and $|\tilde{z}^k|$ limits. Taking the $|z^i| \to \infty$ limit and considering the finite region on the z-plane, (3.8) reduces to

$$P = \begin{pmatrix} 0 & 0 \\ 0 & 1 - \sum_{k,l=1}^{s} |\tilde{z}^k\rangle \tilde{h}^{-1}_{kl} \langle \tilde{z}^l| \end{pmatrix} . \tag{3.9}$$

This is the BPS anti-soliton solution. On the other hand, in the limit of $|\tilde{z}^k| \to \infty$, (3.8) reduces to

$$P = \begin{pmatrix} \sum_{i,j=1}^{r} |z^i\rangle h_{ij}^{-1} \langle z^j| & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.10}$$

This is the BPS soliton solution. Thus we can interpret the non-BPS solution (3.8) as the configuration which contains r solitons at $z = z^1, \ldots, z^r$ and s anti-solitons at $z = \tilde{z}^1, \ldots, \tilde{z}^s$. The topological charge and the energy of the solution (3.8) are the sums of the contributions of P_1 and P_2. Since the contributions of P_1 and P_2 to the topological charge are r and $-s$ respectively, we obtain $Q = r - s$ and $E = 2\pi(r + s)$.

3.2 Stability

We analyze the stability of the solution which contains one soliton at z and one anti-soliton at $z = 0$

$$P = \begin{pmatrix} |z\rangle\langle z| & 0 \\ 0 & 1 - |0\rangle\langle 0| \end{pmatrix}. \tag{3.11}$$

We connect this solution to the vacuum solution

$$P_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.12}$$

by a path

$$P_\phi = \begin{pmatrix} \sin^2\phi |z\rangle\langle z| & \sin\phi\cos\phi |z\rangle\langle 0| \\ \sin\phi\cos\phi |0\rangle\langle z| & 1 - \sin^2\phi |0\rangle\langle 0| \end{pmatrix}, \qquad \phi \in \left[0, \frac{\pi}{2}\right], \tag{3.13}$$

which gives P_0 at $\phi = 0$ and P at $\phi = \frac{\pi}{2}$. The energy of the configuration (3.13) is

$$E = 2\pi\theta \operatorname{Tr}(\partial_{\bar{z}} P_\phi \partial_z P_\phi) = 4\pi \sin^2\phi \left(1 + \frac{\bar{z}z}{\theta}\cos^2\phi\right). \tag{3.14}$$

Differentiating this with respect to ϕ twice we obtain

$$\left.\frac{\partial^2 E}{\partial \phi^2}\right|_{\phi=\frac{\pi}{2}} = 8\pi\left(\frac{\bar{z}z}{\theta} - 1\right). \tag{3.15}$$

The energy (3.14) has a minimum at $\phi = 0$. When $\bar{z}z < \theta$ the energy (3.14) has a local maximum at $\phi = \pi/2$ and monotonically decreases to zero at $\phi = 0$. In this case the solution (3.11) is unstable and the soliton anti-soliton pair annihilates. When $\bar{z}z > \theta$ the energy (3.14) has a local minimum at $\phi = \pi/2$ and therefore the solution (3.11) is metastable in this parameter space. We do not know whether the solution is unstable under fluctuations in other directions.

3.3 Time-dependent solution

Time-dependent solutions can be obtained by boosting static solutions $P(\hat{z}, \hat{\bar{z}})$. The solution moving in the $x(= \sqrt{2}\operatorname{Re} z)$-direction with the velocity v is given by

$$P_v(\hat{z}, \hat{\bar{z}}, t) = P(\hat{z}', \hat{\bar{z}}'), \tag{3.16}$$

which satisfies the equation of motion

$$[\partial_t^2 P_v - 2\partial_{\bar z}\partial_z P_v, P_v] = 0\,, \tag{3.17}$$

where $\hat z' = \frac{1}{\sqrt2}(\hat x' + i\hat y')$, $\hat{\bar z}' = \frac{1}{\sqrt2}(\hat x' - i\hat y')$ are given by the Lorentz transformation

$$\hat t' = \frac{t - v\hat x}{\sqrt{1-v^2}}\,, \qquad \hat x' = \frac{\hat x - vt}{\sqrt{1-v^2}}\,, \qquad \hat y' = \hat y\,. \tag{3.18}$$

The spatial coordinates $\hat z'$, $\hat{\bar z}'$ obey

$$[\hat z', \hat{\bar z}'] = \theta'\,, \qquad \theta' = \frac{\theta}{\sqrt{1-v^2}}\,. \tag{3.19}$$

The Lorentz symmetry is explicitly broken by the noncommutativity. The moving solution is obtained by the boost accompanied by rescaling of the noncommutative parameter [14].

For the solution of the diagonal form (3.1), P_1 and P_2 can be boosted with the different velocities $\vec v_1$ and $\vec v_2$ respectively. We introduce the coordinates $\hat z_1(\hat z_2)$, $\hat{\bar z}_1(\hat{\bar z}_2)$ given by the Lorentz transformation with the velocity $\vec v_1(\vec v_2)$. These coordinates obey the same commutation relation as (3.19)

$$[\hat z_a, \hat{\bar z}_a] = \theta_a\,, \qquad \theta_a = \frac{\theta}{\sqrt{1-v_a^2}}\,, \qquad a = 1, 2\,. \tag{3.20}$$

Boosting the solution (3.8) we obtain the time-dependent solution

$$P_{v_1 v_2} = \begin{pmatrix} \sum_{i,j=1}^r |z_1^i\rangle h_{1,ij}^{-1} \langle z_1^j| & 0 \\ 0 & 1 - \sum_{k,l=1}^s |z_2^k\rangle h_{2,kl}^{-1}\langle z_2^l| \end{pmatrix}, \tag{3.21}$$

where

$$\begin{aligned} |z_a^i\rangle &= e^{{\theta_a}^{-1}(z_a^i \hat{\bar z}_a - \bar z_a^i \hat z_a)}|0_a\rangle\,, \\ \hat z_a|0_a\rangle &= 0\,, \qquad\qquad \langle 0_a|0_a\rangle = 1\,, \\ h_a^{ij} &= \langle z_a^i | z_a^j\rangle\,, \qquad\qquad a = 1, 2\,. \end{aligned} \tag{3.22}$$

When $r = 1$, $s = 1$ the solution (3.21) represents a soliton and an anti-soliton moving independently with arbitrary velocities. In the case of $r > 1$ ($s > 1$) the energy density of the solution (3.21) may have multiple peaks, but this solution is not a time-dependent multi-(anti-)soliton configuration where (anti-)soliton peaks display relative motion because all (anti-)soliton peaks move with a common velocity.

4. Other solutions

4.1 Other non-BPS solutions

We can construct other non-BPS solutions of the form (3.1) by taking the diagonal elements P_1 and P_2 to be solutions of the equation of motion which do not satisfy the BPS equations. The operator $\sum_{i=1}^k |n_i\rangle\langle n_i|$ $(0 \le n_1 < \ldots < n_k)$ satisfies the equation of motion (2.16) but

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not the BPS equations (2.17) nor (2.18) except for the case of $n_i = i-1$, $i = 1, \ldots, k$ where this operator is the solution of the BPS equation (2.17). For example, choosing $P_1 = |n\rangle\langle n|$ $(n > 0)$ we have the non-BPS solution

$$P = \begin{pmatrix} |n\rangle\langle n| & 0 \\ 0 & 1 \end{pmatrix}, \qquad n > 0\,. \tag{4.1}$$

This configuration has the topological charge $Q = 1$ and the energy $E = 2\pi(2n+1)$.

We can also consider the non-BPS solution

$$P = \begin{pmatrix} |n\rangle\langle n| & 0 \\ 0 & |m\rangle\langle m| \end{pmatrix}, \qquad n > 0,\ m \geq 0\,. \tag{4.2}$$

For this solution we cannot find W from (2.10). This is the solution of (2.16) but not a solution of the equation of motion derived from the lagrangian (2.2).

4.2 Finite size solution

We have constructed the solutions which have the zero size in the commutative limit. In this subsection we consider the finite size solution which has a parameter μ related to the size of the solution. As mentioned in section 2, $\tilde{W} = \begin{pmatrix} \mu\hat{z}^{-1} \\ 1 \end{pmatrix}$ is not a solution. The projection operator

$$\tilde{P} = \tilde{W}\frac{1}{\tilde{W}^\dagger\tilde{W}}\tilde{W}^\dagger = \begin{pmatrix} \hat{\bar{z}}\frac{\mu^2}{(\hat{\bar{z}}\hat{z}+\theta)(\mu^2+\hat{\bar{z}}\hat{z}+\theta)}\hat{z} & \hat{\bar{z}}\frac{\mu}{\mu^2+\hat{\bar{z}}\hat{z}+\theta} \\ \frac{\mu}{\mu^2+\hat{\bar{z}}\hat{z}+\theta}\hat{z} & \frac{1}{\mu^2+\hat{\bar{z}}\hat{z}+\theta}(\hat{\bar{z}}\hat{z}+\theta) \end{pmatrix} \tag{4.3}$$

does not satisfy (2.16), (2.17) nor (2.18). We show that we can construct the solution by adding the correction to $\tilde{P}$. We consider the projection operator

$$P = \tilde{P} + \frac{1}{\mu^2+\theta}\begin{pmatrix} \theta|1\rangle\langle 1| & -\mu\sqrt{\theta}|1\rangle\langle 0| \\ -\mu\sqrt{\theta}|0\rangle\langle 1| & \mu^2|0\rangle\langle 0| \end{pmatrix}. \tag{4.4}$$

After a little algebra we obtain

$$(1-P)\hat{z}P = \begin{pmatrix} \sqrt{\theta}|0\rangle\langle 1| & 0 \\ 0 & 0 \end{pmatrix}. \tag{4.5}$$

This implies that P is not a BPS soliton solution but satisfies the equation of motion. Calculating W which generates P, we obtain

$$W = \begin{pmatrix} \mu\hat{z}^{-1} \\ 1 \end{pmatrix}\hat{z} + c\begin{pmatrix} 1 \\ -\mu\hat{\bar{z}}^{-1} \end{pmatrix}|1\rangle\langle 0| = \begin{pmatrix} \mu(1-|0\rangle\langle 0|) + c|1\rangle\langle 0| \\ \hat{z} - c\frac{\mu}{\sqrt{\theta}}|0\rangle\langle 0| \end{pmatrix}, \tag{4.6}$$

where c is a constant. The topological charge is

$$\begin{aligned} Q &= \theta\,\mathrm{Tr}\left[\frac{1}{W^\dagger W}\left((\partial_z W)^\dagger(1-P)\partial_z W - (\partial_{\bar{z}} W)^\dagger(1-P)\partial_{\bar{z}} W\right)\right] \\ &= \theta\,\mathrm{Tr}\left[\frac{\mu^2}{\mu^2+\hat{\bar{z}}\hat{z}}\left(\frac{1}{\theta}|0\rangle\langle 0| + \frac{1}{\mu^2+\hat{\bar{z}}\hat{z}+\theta} - \left(\frac{1}{\theta}|1\rangle\langle 1| + \frac{1}{\mu^2+\theta}|0\rangle\langle 0|\right)\right)\right] \\ &= \theta\left[\frac{1}{\theta} + \sum_{n=0}^{\infty}\frac{\mu^2}{(\mu^2+\theta n)(\mu^2+\theta n+\theta)} - \left(\frac{\mu^2}{\theta(\mu^2+\theta)} + \frac{1}{\mu^2+\theta}\right)\right] \\ &= 1+1-1 = 1\,, \end{aligned} \tag{4.7}$$

where we have set $c = \sqrt{\frac{\mu^2\theta}{\mu^2+\theta}}$ for simplicity because Q is independent of c. The energy of this configuration is $E = 2\pi(2+1) = 6\pi$.

The solution (4.4) has the parameter μ of the dimension of length. To see that this parameter is related to the size of the solution, we consider the small μ and large μ limits. In the limit of $\mu \to 0$, (4.4) reduces to

$$P = \begin{pmatrix} |1\rangle\langle 1| & 0 \\ 0 & 1 \end{pmatrix}. \tag{4.8}$$

This corresponds to (4.1) with $n = 1$. On the other hand, in the limit of $\mu \to \infty$, (4.4) reduces to

$$P = \begin{pmatrix} 1 - |0\rangle\langle 0| & 0 \\ 0 & |0\rangle\langle 0| \end{pmatrix}. \tag{4.9}$$

This corresponds to the solution considered in section 3 and represents a soliton anti-soliton pair sitting at the origin. We can interpret the non-BPS solution (4.4) as the configuration which contains a soliton of the size μ and a small soliton anti-soliton pair. In the large μ limit the soliton spreads over the space and disappears, and hence only the soliton anti-soliton pair exists.

5. Discussion

In this paper, we have constructed non-BPS solutions of the noncommutative CP^1 model. These solutions can be extended to the case of CP^N, $N \geq 2$ by embedding the solutions in the $(N+1) \times (N+1)$ projection operator of the block diagonal form. In the commutative CP^N model, all the static non-BPS solutions were generated from the BPS solutions [12]. It is a future problem whether we can construct all static solutions of the noncommutative CP^N model. In noncommutative gauge theories, (non-)BPS solutions were constructed by using the solution generating transformation [2]. It might be possible to find such a trasformation in the noncommutative CP^N model.

We have a comment on the relation between the noncommutative CP^N model and a scalar theory on a noncommutative space (the GMS model) [4]. Our non-BPS solutions have been constructed by using solitons in the GMS model. If one considers the $(N + 1) \times (N + 1)$ hermitian matrix instead of the scalar field in the GMS model, one gets the noncommutative CP^N model in the low-energy limit.

Some non-Bogomol'nyi solutions of the Yang-Mills-Higgs equations were obtained by using the non-BPS solutions of the CP^N model [15]. It is interesting to see whether our non-BPS solutions can be used to construct new non-Bogomol'nyi solutions of the Yang-Mills-Higgs system on a noncommutative space [16].

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References

[1] M.R. Douglas and N.A. Nekrasov, *Noncommutative field theory*, *Rev. Mod. Phys.* **73** (2002) 977.

[2] J.A. Harvey, *Komaba lectures on noncommutative solitons and D-branes*, `hep-th/0102076`.

[3] N. Nekrasov and A. Schwarz, *Instantons on noncommutative R^4 and (2,0) superconformal six dimensional theory*, *Commun. Math. Phys.* **198** (1998) 689 [`hep-th/9802068`].

[4] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020.

[5] M. Hamanaka, Y. Imaizumi and N. Ohta, *Moduli space and scattering of D0-branes in noncommutative super Yang-Mills theory*, *Phys. Lett.* **B 529** (2002) 163 [`hep-th/0112050`].

[6] R. Gopakumar, M. Headrick and M. Spradlin, *On noncommutative multi-solitons*, `hep-th/0103256`.

[7] U. Lindström, M. Roček and R. von Unge, *Non-commutative soliton scattering J. High Energy Phys.* **12** (2000) 004;
T. Araki and K. Ito, *Scattering of noncommutative (N,1) solitons*, *Phys. Lett.* **B 516** (2001) 123.

[8] B.-H. Lee, K.-M. Lee and H.S. Yang, *The CP(n) model on noncommutative plane*, *Phys. Lett.* **B 498** (2001) 277 [`hep-th/0007140`].

[9] K. Furuta, T. Inami, H. Nakajima and M. Yamamoto, *Low-energy dynamics of noncommutative CP^1 solitons in $2+1$ dimensions*, *Phys. Lett.* **B 537** (2002) 165 [`hep-th/0203125`].

[10] O. Lechtenfeld and A.D. Popov, *Noncommutative multi-solitons in $2+1$ dimensions*, *J. High Energy Phys.* **11** (2001) 040 [`hep-th/0106213`]; *Scattering of noncommutative solitons in $2+1$ dimensions*, *Phys. Lett.* **B 523** (2001) 178 [`hep-th/0108118`].

[11] M. Wolf, *Soliton antisoliton scattering configurations in a noncommutative sigma model in $2+1$ dimensions*, *J. High Energy Phys.* **06** (2002) 055 [`hep-th/0204185`].

[12] A.M. Din and W.J. Zakrzewski, *General classical solutions in the $CP^{(n-1)}$ model*, *Nucl. Phys.* **B 174** (1980) 397.

[13] L. Hadasz, U. Lindström, M. Roček and R. von Unge, *Noncommutative multisolitons: moduli spaces, quantization, finite theta effects and stability*, *J. High Energy Phys.* **06** (2001) 040 [`hep-th/0104017`].

[14] D. Bak and K.-M. Lee, *Elongation of moving noncommutative solitons*, *Phys. Lett.* **B 495** (2000) 231 [`hep-th/0007107`].

[15] T.A. Ioannidou and P.M. Sutcliffe, *Non-bogomolny* $\mathrm{SU}(N)$ *BPS monopoles*, *Phys. Rev.* **D 60** (1999) 105009 [`hep-th/9905169`].

[16] D.J. Gross and N.A. Nekrasov, *Solitons in noncommutative gauge theory*, *J. High Energy Phys.* **03** (2001) 044 [`hep-th/0010090`].